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Gamma-Delta Connectedness as a Characterization of Topological Space

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Abstract: In this paper, we discuss the basic properties of γ_Δ-Os and γ_Δ-relatedness. Topology arose as a branch of study from geometry and predicate logic, based on the examination of notions such as spatial dimension and modification.

1. Introduction

The term topology was used by Johann Bebedict in the nineteenth century, but the concept of a topological space (TS) was not established until another decade of the twentieth century.

The investigation of connectivity through various open set (Os) is not a novel concept in Ts. Njastad developed the gamma - Os in 1963 and studied the topology on the class of such sets; the gamma - Os create a topology.

Levine (1963), A.H.Mashhour (1982), and Andrijevic (1986) investigated the categories of quasi sets, pre-Os , and Delta - Os , accordingly.

The class of Os is enclosed in each of the classes of Delta -open, gamma -open, pre-open, and semi- Os. The notions of Delta-relatedness, gamma-relatedness, pre-relatedness, and quasi were introduced based on the classes of Delta-Os, gamma - Os, which was before sets, and semi- Os. It is well understood that the relatedness of a TS Y is identical to the gamma -relatedness of Y.

The γ_Δ - Os is introduced in a TS Y which forms a topology on Y and on X, but it is enclosed in the topology of γ - Os. The relatedness of this new topology on X, called γ_Δ -relatedness of Y is equivalent to relatedness of the original TS & Y and hence to γ-relatedness of X.

2. Preliminaries

Definition 2.1:

A topology happening and locate Y is a group λ of subsets of Y have the below conditions.

(1) Ψ & Y belongs to λ.

(2) The combination of basics of any associate-collection of λ is in λ.

(3) The connection of the basics of any limited associate-collection of λ is in λ . A set Y is a topology λ havebe particular is denoted a TS .

Definition 2.2:

If Y is a TS with λ , and it is denoted as subset V of Y is an open set of Y , if V is the depends on λ .

Definition 2.3:

Let A of a TS Y is state that closed, if the set $Y - B$ is open.

Definition 2.4:

Let B of a TS Y. The interior of B is defined as the union of all Os enclosed in B and is represent by Int.(B) .

Definition 2.5:

Given a subset B of a TS X. The closure of B is defined as the intersection of all Cs

enclosing B and is represent by Cz(B).

Definition 2.6:

Let B of TS of Y is denoted as,

(1) γ -open, if $B\subseteq$ Int.(Cz(Int(B))).

(2) Δ -open, if $B\subseteq$ Cz(Int.(Cz(B))).

(3) Pre-open, if $B\subseteq$ Int.(Cz(B)).

(4) Semi-open, if $B\subseteq$ Cz(Int.(B)).

The relations of all,

(i) γ -open subsets of Y is representing by $\gamma R(X)$.
(ii) Δ -open subsets of Y is representing by $\Delta R(X)$.

- The accompaniment of γ - Os is state that γ -closed.
- The accompaniment of Δ -Os is state that Δ -closed.

Let consider to all,

(iii) γ -Cs of Y is representing by $\gamma C(X)$.

(iv) Δ -Cs of Y is representing by $\Delta C(X)$.

Definition 2.7:

A position $x \in X$ is state that a γ_Δ -inner position of $B \subseteq X$, if there endure a γ_Δ -Os V enclosing Y it is belongs to $x \in U \subseteq A$. To all the set γ_Δ - inner position of B state that $\boldsymbol{\gamma_\Delta}$ -inner of B and is represent by γ_ΔInt.(B) .

Definition 2.8:

A point $y \in Y$ is state that a Q_Δ -inner position of $B \subseteq X$, if here endure a Q_Δ -Os V enclosing Y . It is denoted as $x \in U \subseteq A$.
Let Q_Δ - inner position of B is state that Q_Δ -inner of B and is represent by Q_ΔInt.(A) .

Definition 2.9:

(1) The intersection of all γ_Δ - Cs enclosing a subset G of Y will be called

$\boldsymbol{\gamma_\Delta}$ -closure of G and will be represent by $\gamma_\Delta Cl(F)$.

(2) The intersection of all Q_Δ - Cs enclosing a subset G of Y will be called

Q_Δ -closure of G and will be represent by $Q_\Delta Cl(F)$.

Definition 2.10:

The B is subset of TS Y is state that dense in Y , if $\overline{B} = Y$.

Definition 2.11:

A TS (Y, λ) is state that

(1) Close by indiscrete, if all Os of *Y* is closed.

(2) Hyper-related, if all non-unfilled Os of Y is solid in X.

(3) Extremely dissertated, if $Cz(V) \in \lambda$ for each $V \in \lambda$.

(4) Close by pre-indiscrete, if all pre-Os of Y is closed.

(5) Close by γ -indiscrete, if every γ -Os of Y is closed.

Definition 2.12:

Let us consider X and Y be two Ts. A map $g: X \to Y$ is state that

(1). A closed map, for each Cs B of X, which is closed ser $g(A)$ in Y .
(2). An open map, of each open set V of X, the set $g(U)$ is open in Y .

Definition 2.13:

Let X and Y be two Ts. A function $g: X \to Y$ is state that continuous, if for each Os V of Y, the set $g^{-1}(V)$ is an Os of X.

Definition 2.14:

Let us consider Y *be* a TS. A **partition** of Y *is* a couple U, V *of* put out of articulation nonempty Os of Y whose combination is X. The space Y is state that **related**, if is not valid a partition of X.

Definition 2.15:

Given X, describe an sameness relation on Y by setting $x \times y$ if there is a related subspace of Y enclosing both Y and y. The sameness classes are called the **mechanism** of X.

Definition 2.16:

A space Y *is* state that near related at x, if for all region V of x, there is a related region V of Y enclosed in U.
If Y is close by related at every of its points, then it is said simply to be near related.

3. γ_Δ-Related Spaces

3.1 Properties of γ_Δ -related sets:

The γ_Δ -closure of a subset B in a space Y may be distinct from the closure of B, but they are related through relatedness, that is both types of closure are related as well as γ_Δ -related.

Since the relatedness is equivalent to γ_Δ -relatedness, the following properties in this section follow and are induced for the sake of complete us considereness only.

Definition 3.1:

The two non-empty subsets A and B of a TS Y are state that

(1) $\boldsymbol{Q_\Delta}$**-separated**, if $B \cap P_\Delta Cz(B) = \varphi = P_\Delta Cz(A) \cap B$.

(2) $\boldsymbol{\gamma_\Delta}$**-separated**, if $B \cap \gamma_\Delta Cz(B) = \varphi = \gamma_\Delta Cz(A) \cap B$.

Definition 3.2:

The S is the subset of a TS of Y is state that

(1) $\boldsymbol{Q_\Delta}$ **-related** in X, if S is not the blending of two Q_Δ - estranged sets in X.
(2) $\boldsymbol{\gamma_\Delta}$ **-related** in X, if S is not the blending of two γ_Δ -estranged sets in X.

Theorem 3.1:

A TS Y is γ_Δ -related IFF Y cannot be expressed as the union of two put out of articulation nonempty γ_Δ -Os of X.

Proof:

Let us consider Y be a γ_Δ -related space.

Let us consider B and B be two put out of articulation nonempty γ_Δ -Os of Y such that $X = A \cup B$. Then both B and B are γ_Δ -closed in X. Thus, $B \cap \gamma_\Delta Cl(B) = \varphi =$ $\gamma_\Delta Cl(A) \cap B$.
Then Y is not γ_Δ -related, which is a contradiction.

$\therefore$ $X f= A \cup B$.

i. e, Y cannot be expressed as the union of two disjoint nonempty γ_Δ-open subsets of X .

Lemma 3.1:

A nonempty proper subset B of a TS Y is both γ_Δ -open and γ_Δ -closed IFF B is both open and closed.

Proof:

Let us consider B be a nonempty proper subset of the TS X. Assume that B is both γ_Δ -open and γ_Δ -closed.

Then B is both γ -open and γ -closed in X, *and* $B = X - A$ is also both γ_Δ -open and γ_Δ -closed.

Then $Cl(Int.(A)) \subsetneq Cz(B)$ implies that $Int.(Cz(Int.(B))) \subsetneq Int(Cl(A)) \subsetneq$

B , since B is γ -closed and hence semi-closed.

Then we get $B = Int.(Cz(Int(B)))$ which means that B is open. Similarly B is open.
$\Rightarrow X - B = A$ is closed.

Thus B is both open and closed in X.

Converse part:

Assume that B is both open and closed in X. Then B is γ -open and Δ -closed.
$=\Rightarrow$ B is γ_Δ -open.

Similarly, $Y - B$ is γ_Δ-open.

$=\Rightarrow$ B is γ_Δ -closed.

Thus B is both γ_Δ -open and γ_Δ -closed.

Theorem 3.2:

Let us consider TS X, the below are equal. (1).Then Y is γ_Δ -related. (2).The subset of Y is merely and which are together γ_Δ-open γ_Δ -closed are Y and the unfilled set. (3).There is no non-steady onto γ_Δ-unremitting function from Y to a discrete gap which encloses extra than one point.

Proof:

(1) =⇒ (2):

Assume that Y is γ_Δ -related.

Let us consider B and B be two put out of articulation nonempty γ_Δ -open subsets of X .
If $B \cup B = X$, then B and B are γ_Δ -closed in X . Then by the theorem "A TS Y is γ_Δ -related if Y cannot be expressed as the combination of
two put out of articulation nonempty γ_Δ -open subsets of X ." which is a contradiction to $B \cup B = X$.
$\therefore$ $X f= A \cup B$.

Hence, the only subsets of Y which are both γ_Δ -open and γ_Δ -closed are Y

and the empty set.

(2) =⇒ (3):

Assume that condition (2) holds. To prove condition (3),
Let us consider Y be a discrete space with more than one point. Let us consider $g : X \longrightarrow Y$ be an γ_Δ -continuous function.

Let us consider $y \in Y$ and $B = \{y\}$.

Since $g : X \to Y$ is γ_Δ -continuous and onto, then by Lemma 2.1, $g^{-1}(A)$ is nonempty, γ_Δ -open and γ_Δ -closed in X .

Since $g^{-1}(A)$ is nonempty, $g^{-1}(A) = X$. That is, G is constant.
(3) $=\Rightarrow$ (1):

Assume that condition (3) holds. i.e, G is constant.
To prove Y is γ_Δ -related. Suppose that Y is not γ_Δ -related.
If $X = A \cup B$, where B and B are nonempty subsets of Y such that

$\gamma_\Delta Cl(A) \cap B = \varphi = A \cap \gamma_\Delta Cl(B)$, then both B and B are γ_Δ - Os in X .

Assume that $Y = \{0, 1\}$ with the disconnected topology.

To define a map G from Y to Y by $g(x) = 0$, if $x \in A$ and $g(x) = 1$, if $x \in B$.

The G is a non-constant γ_Δ -continuous and onto mapping, which is a opposition to our hypothesis that G is constant.

∴ X is γ_Δ-related.

Theorem 3.3:

If a space Y is hyper-related, then it is γ_Δ -related.

Proof:

Let us consider Y be a hyper-related space. i.e, Every nonempty Os of Y is dense in X .
we have the only possible subsets of Y that are both γ_Δ -open and γ_Δ -closed are Y and φ .

we get that Y is γ_Δ -related.

Theorem 3.4:

Let us consider λ_1 and λ_2 be two topologies on Y with $\lambda_2 \subsetneq \lambda_1$. If (X, λ_1) is γ_Δ -related, then (X, λ_2) is also γ_Δ-related.

Proof:

Let us consider λ_1 and λ_2 be two topologies on Y with $\lambda_2 \subsetneq \lambda_1$ and (X, λ_1) be γ_Δ -related.
To prove (X, λ_2) is γ_Δ -related, Suppose that (X, λ_2) is not γ_Δ -related.
Then there are disjoint nonempty subsets B and B , which are γ_Δ -open in (X, λ_2) with $X = A \cup B$.

we have, B and B are cl Os in (X, λ_2) as B and B are both γ_Δ -open and γ_Δ - Cs in (X, λ_2) .
Therefore B and B are cl Os in (X, λ_1) .

This means that B and B are nonempty γ_Δ - Os in (X, λ_1) , which is a contradiction to (X, λ_1) is γ_Δ -related.
∴ (X, λ_2) is γ_Δ -related.

Theorem 3.5:

A TS Y is γ_Δ -related iff Y is related.

Proof:

Assume that Y is γ_Δ -related. To prove Y is related,
Let us consider B be a nonempty proper subset of Y which is both open And closed in X .

Then B is both γ_Δ -open and γ_Δ -closed. Since Y is γ_Δ -related, and then the only subsets of Y which are both γ_Δ -open and γ_Δ -closed are Y and the empty set.

Hence Y and Ψ are both open and closed.

X is related.

Converse part:

Assume that Y is related. To prove Y is γ_Δ -related, By known theorem,
"The only subsets of a related space Y are open and closed are Y and the unfilled set."
we have Y and Ψ are open and closed.

Then B is both γ_Δ -open and γ_Δ -closed.

By Theorem 2.2, Y is γ_Δ -related.

Corollary 3.2:

A nonempty subset Y of the factual line through the common topology is

γ_Δ -related IFF it is an interval. In fussy, the real line is γ_Δ -related.

Proof:

We know that the real line R is related. Then by Theorem 2.5,
"A TS Y is γ_Δ -related IFF Y is related."

$\therefore$ R is γ_Δ -related.

Theorem 3.6:

$\gamma_\Delta O(X)$ forms a topology on X .

Proof:

Let us consider $\gamma_\Delta O(X)$ be the family of all γ_Δ - Os in X . It is clear that $\gamma_\Delta O(X)$ is non-empty.
To prove $\gamma_\Delta O(X)$ forms a topology on X , It must satisfy the properties of topology.

(1) Clearly, $\varphi, X \in \gamma_\Delta O(X)$.

(2) Let us consider $B_i \in \gamma_\Delta O(X)$ for $i \in I$, and $x \in \cup A_i$. Then, $x \in A_i$ for some $i \in I$.
So there endure a Δ - Cs G such that $x \in F \subseteq A_i$. Then, $x \in F \subseteq \cup A_i$.

(3) Let us consider $K, L \in \gamma_\Delta O(X)$ and $x \in A \cap B$.

Then there are Δ - Cs g_1 and g_2 such that $x \in F_1 \subseteq A$ and $x \in F_2 \subseteq B$.

$\therefore$ $F_1 \cap F_2$ is a Δ-Cs such that $x \in F_1 \cap F_2 \subseteq A \cap B$. Also, $B \cap B \in \gamma_\Delta O(X)$, since $K, L \in \gamma O(X)$.
$\therefore$ $\gamma_\Delta O(X)$ satisfies the properties of topology. Thus $\gamma_\Delta O(X)$ forms a topology on X .

Theorem 3.7:

Let us consider Y be a nearby pre-indiscrete space. Then Y is pre-related IFF it is γ_Δ -related.

Proof:

Given, Y is a nearby pre-indiscrete space. Assume that Y is pre-related.
Then by theorem,

"Let us consider Y be a near pre-indiscrete space. Then Y is pre-related IFF it is related."
We have, Y is related. Then by theorem,
"A TS Y is γ_Δ -related IFF Y is related."

$\therefore$ X is γ_Δ-related.

Converse part:

Assume that Y is γ_Δ -related. Then by theorem,
"A TS Y is γ_Δ -related IFF Y is related." We have, Y is related.
Then by theorem,

"Let us consider Y be a nearby pre-indiscrete space. Then Y is pre-related IFF it is related."
$\therefore$ X is pre-related.

Theorem 3.8:

Let us consider Y be a near pre-indiscrete space. Then Y is semi-related IFF it is γ_Δ -related.

Proof:

Given, Y is a near pre-indiscrete space. Assume that Y is semi-related.
Then by theorem,

"Let us consider Y be a near pre-indiscrete space. Then Y *is* pre-related IFF it is semi-related."
we have Y is pre-related. Then by theorem,
"Let us consider Y be a near pre-indiscrete space. Then Y is pre-related IFF it is γ_Δ -related."
we have Y is γ_Δ -related.

Converse part:

Assume that Y is γ_Δ -related. Then by theorem,
"Let us consider Y be a near pre-indiscrete space. Then Y is pre-related IFF it is γ_Δ -related."
we have Y is pre-related. Then by theorem,
"Let us consider Y be a near pre-indiscrete space. Then Y *is* pre-related IFF it is semi-related."
we have Y is semi-related.

Theorem 3.9:
Let us consider Y be a near γ -indiscrete space. If Y is
(1) Pre-related
(2) Q_Δ-related
(3) Semi-related
(4) Δ-related
(5) γ -related
(6) γ_Δ-related

space, then $\gamma O(X) = \{\varphi, X\}$.

Proof:

Given, Y is a near γ -indiscrete space. i.e, Every γ -Os of Y is closed.

"In a near γ -indiscrete space, every γ -open set is clopen." we have every γ -open set in Y is both open and closed.
By the result,

"Let us consider Y be a near γ -indiscrete space. Then,

(1) $\gamma O(X) \subsetneq PO(X) \cap PC(X)$.

(2) $\gamma O(X) \subsetneq SO(X) \cap SC(X)$

(3) $\gamma O(X) \subsetneq \Delta O(X) \cap \Delta C(X)$
(4) $\gamma O(X) \subsetneq \gamma_\Delta O(X) \cap \gamma_\Delta C(X)$."

we have each γ-open set of Y is pre-open (semi-open, Δ-open, γ_Δ-open also Q_Δ-open) and pre-closed (resp., semi-closed, Δ-closed, γ_Δ-closed also Q_Δ-closed).

Then by theorem,

"The only subsets of a pre-related (semi-related, Δ-related, γ_Δ-related also Q_Δ -related) space Y which are both pre-open (semi-open, Δ -open, γ_Δ -open also Q_Δ-open) and pre-closed (resp., semi-closed, Δ-closed, γ_Δ-closed also Q_Δ-closed) are Y and the empty set."
we have $\gamma O(X) = \{\varphi, X\}$.

4. Conclusion

In this paper, we discussed the basic properties of γ_Δ -open sets and γ_Δ - relatedness. The geometry and predicate logic verified based on the examination of notions such as spatial dimension and modification. The investigation of connectivity through various open sets is not a novel concept in topological spaces.

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